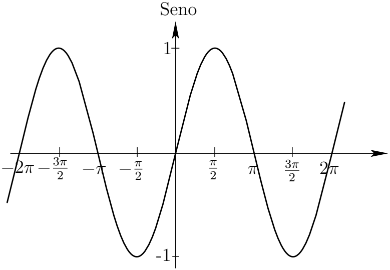
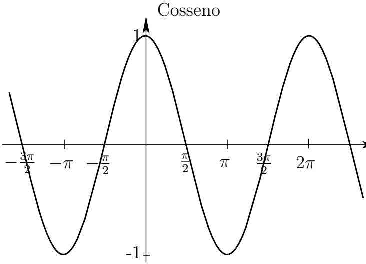
Funções Especiais

Maria Joana Torres

2021/22

Gráficos das funções trigonométricas





Tangente

$$\operatorname{tg} : \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi : \ k \in \mathbb{Z}\right\} \longrightarrow \mathbb{R} \quad \text{tal que} \quad \operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x}$$

Cotangente

$$\operatorname{cotg} : \mathbb{R} \setminus \{k\pi : \ k \in \mathbb{Z}\} \longrightarrow \mathbb{R} \quad \text{tal que} \quad \operatorname{cotg} x = \frac{\cos x}{\operatorname{sen} x}$$

Secante

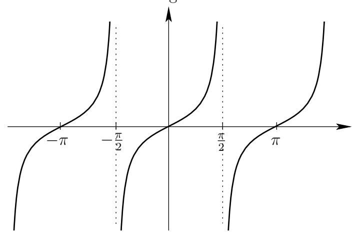
$$\sec : \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi : \ k \in \mathbb{Z}\right\} \longrightarrow \mathbb{R} \quad \text{tal que} \quad \sec x = \frac{1}{\cos x}$$

Cossecante

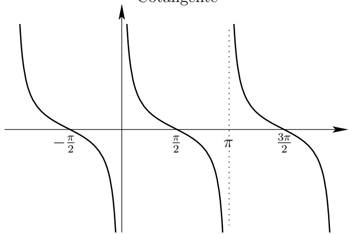
$$\operatorname{cosec} : \mathbb{R} \setminus \{k\pi : \ k \in \mathbb{Z}\} \longrightarrow \mathbb{R} \quad \text{tal que} \quad \operatorname{cosec} x = \frac{1}{\operatorname{sen} x}$$

Gráficos das funções trigonométricas

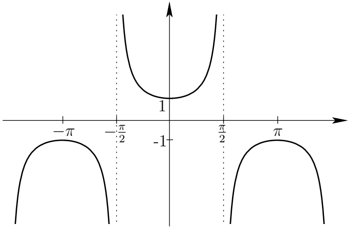
Tangente



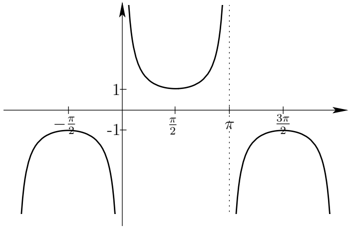
Cotangente



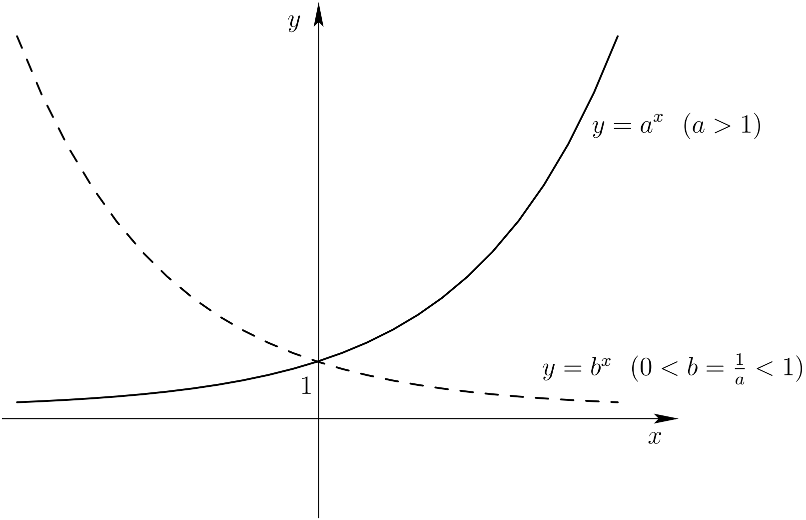
Secante

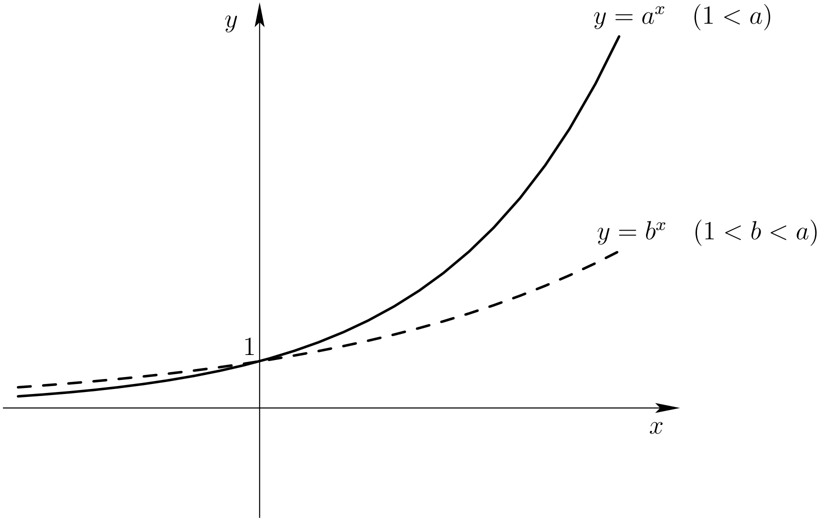


Cossecante

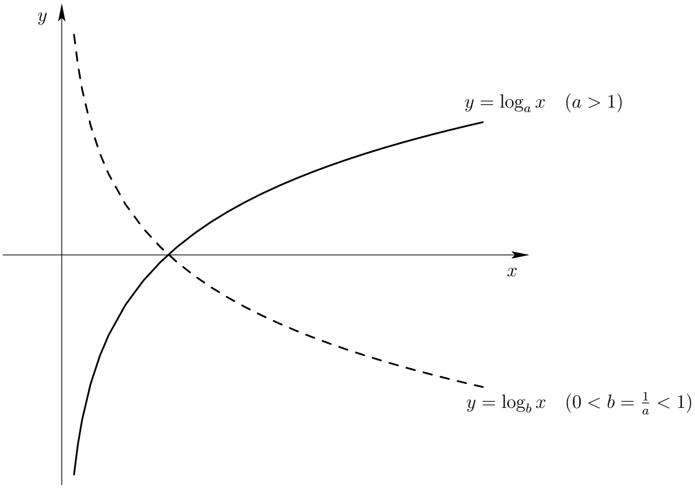


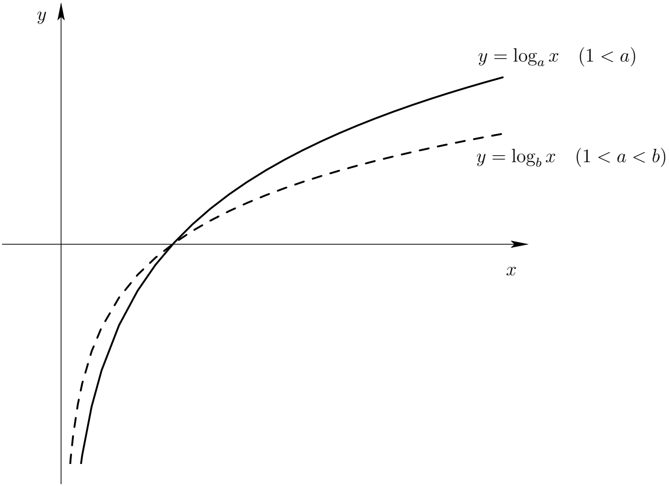
1. $\forall\, a \in \mathbb{R} \quad \operatorname{sen}^2 a + \cos^2 a = 1$;
2. $\forall\, a \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{tg}^2 a = \sec^2 a$;
3. $\forall\, a \in \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{cotg}^2 a = \operatorname{cosec}^2 a$;
4. $\forall\, a \in \mathbb{R} \quad \operatorname{sen}(-a) = -\operatorname{sen} a$ (a função seno é ímpar);
5. $\forall\, a \in \mathbb{R} \quad \cos(-a) = \cos a$ (a função cosseno é par);
6. $\forall\, a \in \mathbb{R} \quad \cos(\frac{\pi}{2} - a) = \operatorname{sen} a$ e $\operatorname{sen}(\frac{\pi}{2} - a) = \cos a$;
7. $\forall\, a \in \mathbb{R} \quad \operatorname{sen}(a + 2\pi) = \operatorname{sen} a$ (a função seno tem período 2π);
8. $\forall\, a \in \mathbb{R} \quad \cos(a + 2\pi) = \cos a$ (a função cosseno tem período 2π);
9. $\forall\, a,\ b \in \mathbb{R} \quad \operatorname{sen}(a + b) = \operatorname{sen} a \cos b + \operatorname{sen} b \cos a$;
10. $\forall\, a,\ b \in \mathbb{R} \quad \cos(a + b) = \cos a \cos b - \operatorname{sen} b \operatorname{sen} a$;
11. $\forall\, a,\ b \in \mathbb{R} \quad \cos a - \cos b = -2 \operatorname{sen} \frac{a-b}{2} \operatorname{sen} \frac{a+b}{2}$;
12. $\forall\, a,\ b \in \mathbb{R} \quad \operatorname{sen} a - \operatorname{sen} b = 2 \operatorname{sen} \frac{a-b}{2} \cos \frac{a+b}{2}$.


y
$y = a^x \quad (a > 1)$
$y = b^x \quad (0 < b = \frac{1}{a} < 1)$
1
x


$y = a^x \quad (1 < a)$
$y = b^x \quad (1 < b < a)$
y
x
1

Funções logaritmos




y
$y = \log_a x \quad (1 < a)$
$y = \log_b x \quad (1 < a < b)$
x

Seno hiperbólico

$$\begin{array}{rccc} \mathrm{sh}: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & \dfrac{e^x - e^{-x}}{2} \end{array}$$

Cosseno hiperbólico

$$\begin{array}{rccc} \mathrm{ch}: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & \dfrac{e^x + e^{-x}}{2} \end{array}$$

Tangente hiperbólica

$$\begin{array}{rccc} \mathrm{th}: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & \dfrac{\mathrm{sh}\, x}{\mathrm{ch}\, x} \end{array}$$

Cotangente hiperbólica

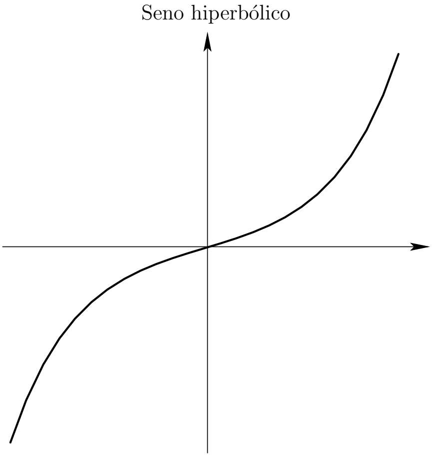
$$\begin{array}{rccc} \mathrm{coth}: & \mathbb{R} \setminus \{0\} & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & \dfrac{1}{\mathrm{th}\, x} \end{array}$$

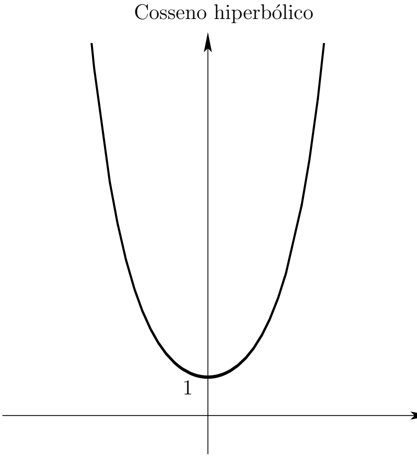
Secante hiperbólica

$$\begin{array}{rccc} \mathrm{sech}: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & \dfrac{1}{\mathrm{ch}\, x} \end{array}$$

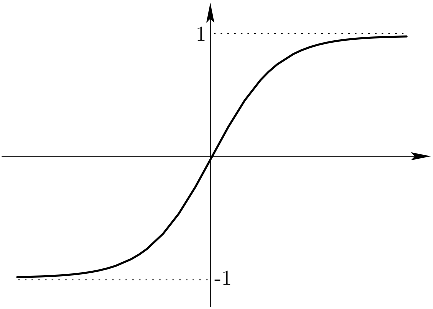
Cossecante hiperbólica

$$\begin{array}{rccc} \mathrm{cosech}: & \mathbb{R} \setminus \{0\} & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & \dfrac{1}{\mathrm{sh}\, x} \end{array}$$

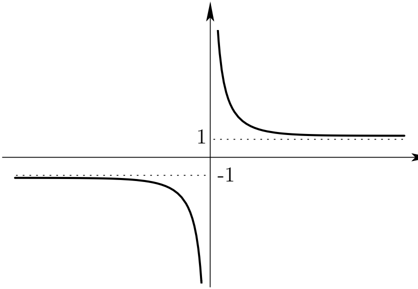

Seno hiperbólico

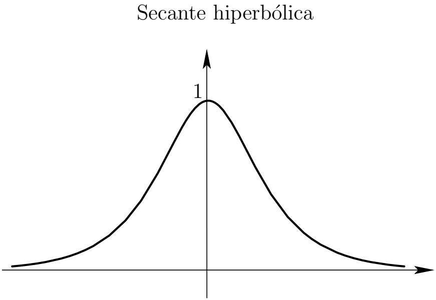

Cosseno hiperbólico
1

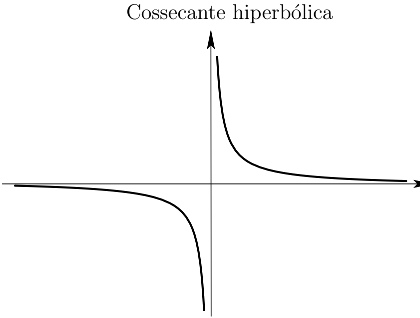
Tangente hiperbólica



Cotangente hiperbólica




Secante hiperbólica
1


Cossecante hiperbólica

1. $\forall a \in \mathbb{R} \qquad \mathrm{ch}^2 a - \mathrm{sh}^2 a = 1;$
2. $\forall a \in \mathbb{R} \qquad \mathrm{th}^2 a + \mathrm{sech}^2 a = 1;$
3. $\forall a \in \mathbb{R} \setminus \{0\} \qquad \mathrm{coth}^2 a - \mathrm{cosech}^2 a = 1;$
4. $\forall a \in \mathbb{R} \qquad \mathrm{sh}\,(-a) = -\mathrm{sh}\, a$ (a função seno hiperbólico é ímpar);
5. $\forall a \in \mathbb{R} \qquad \mathrm{ch}\,(-a) = \mathrm{ch}\, a$ (a função cosseno hiperbólico é par);
6. $\forall a,\, b \in \mathbb{R} \qquad \mathrm{sh}\,(a+b) = \mathrm{sh}\, a\, \mathrm{ch}\, b + \mathrm{sh}\, b\, \mathrm{ch}\, a;$
7. $\forall a,\, b \in \mathbb{R} \qquad \mathrm{ch}\,(a+b) = \mathrm{ch}\, a\, \mathrm{ch}\, b + \mathrm{sh}\, b\, \mathrm{sh}\, a;$
8. $\forall n \in \mathbb{N} \quad \forall a \in \mathbb{R} \qquad (\mathrm{ch}\, a + \mathrm{sh}\, a)^n = \mathrm{ch}\,(na) + \mathrm{sh}\,(na).$

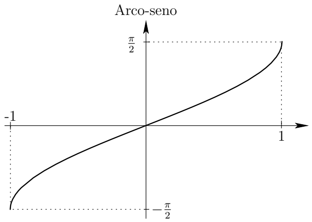
Funções trigonométricas inversas

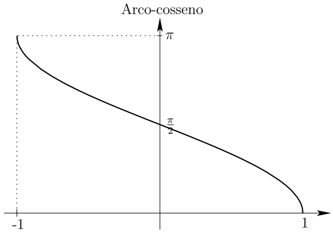
Arco-seno

$$
\begin{array}{rccl}
\operatorname{arcsen}: & [-1,1] & \longrightarrow & \left[-\frac{\pi}{2},\frac{\pi}{2}\right] \\
 & x & \longmapsto & \left(\operatorname{sen}|_{\left[-\frac{\pi}{2},\frac{\pi}{2}\right]}\right)^{-1}(x)
\end{array}
$$

Arco-cosseno

$$
\begin{array}{rccl}
\arccos: & [-1,1] & \longrightarrow & [0,\pi] \\
 & x & \longmapsto & \left(\cos|_{[0,\pi]}\right)^{-1}(x)
\end{array}
$$



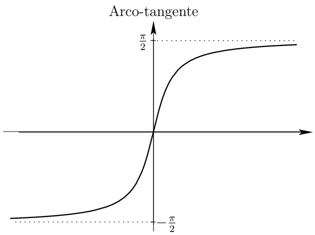


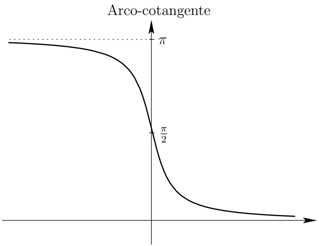
Arco-tangente

$$\begin{array}{rccc} \operatorname{arctg}: & \mathbb{R} & \longrightarrow & \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\\ & x & \longmapsto & \left(\operatorname{tg}|_{\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[}\right)^{-1}(x) \end{array}$$

Arco-cotangente

$$\begin{array}{rccc} \operatorname{arccotg}: & \mathbb{R} & \longrightarrow &]0, \pi[\\ & x & \longmapsto & \left(\operatorname{cotg}|_{]0, \pi[}\right)^{-1}(x) \end{array}$$



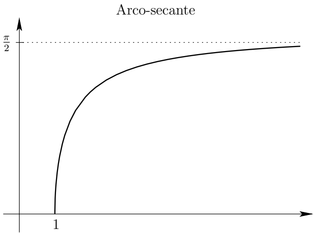


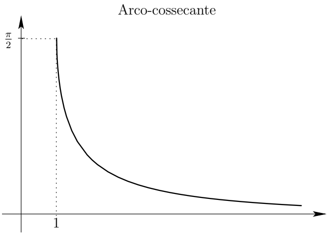
Arco-secante

$$\begin{array}{rccc} \mathrm{arcsec}: & [1,+\infty[& \longrightarrow & [0,\frac{\pi}{2}[\\ & x & \longmapsto & \left(\sec|_{[0,\frac{\pi}{2}[}\right)^{-1}(x) \end{array}$$

Arco-cossecante

$$\begin{array}{rccc} \mathrm{arccosec}: & [1,+\infty[& \longrightarrow &]0,\frac{\pi}{2}] \\ & x & \longmapsto & \left(\mathrm{cosec}|_{]0,\frac{\pi}{2}]}\right)^{-1}(x) \end{array}$$





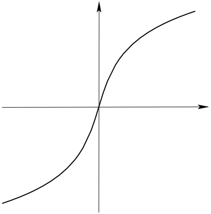
Argumento do seno hiperbólico

$$\begin{array}{rccc} \operatorname{argsh}: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & (\operatorname{sh})^{-1}(x) \end{array}$$

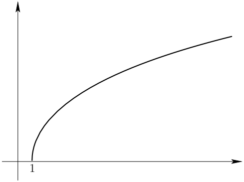
Argumento do cosseno hiperbólico

$$\begin{array}{rccc} \operatorname{argch}: & [1,+\infty[& \longrightarrow & \mathbb{R}_0^+ \\ & x & \longmapsto & \left(\operatorname{ch}_{|_{\mathbb{R}_0^+}}\right)^{-1}(x) \end{array}$$

Argumento do seno hiperbólico



Argumento do cosseno hiperbólico



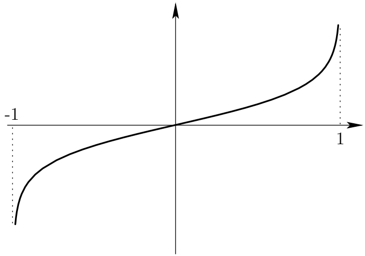
Argumento da tangente hiperbólica

$$\begin{array}{rccc} \text{argth}: &]-1,1[& \longrightarrow & \mathbb{R} \\ & x & \longmapsto & \text{th}^{-1}(x) \end{array}$$

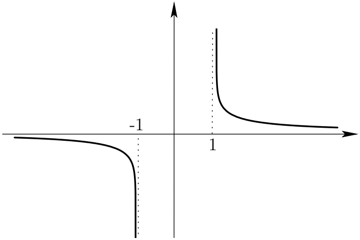
Argumento da cotangente hiperbólica

$$\begin{array}{rccc} \text{argcoth}: & \mathbb{R}\setminus[-1,1] & \longrightarrow & \mathbb{R}\setminus\{0\} \\ & x & \longmapsto & \coth^{-1}(x) \end{array}$$

Argumento da tangente hiperbólica



Argumento da cotangente hiperbólica



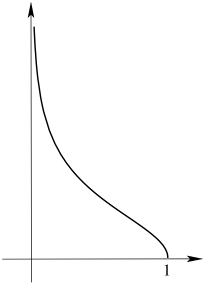
Argumento da secante hiperbólica

$$\begin{array}{rccl} \operatorname{argsech} : &]0,1] & \longrightarrow & \mathbb{R}_0^+ \\ & x & \longmapsto & \left(\sec\big|_{\mathbb{R}_0^+}\right)^{-1}(x) \end{array}$$

Argumento da cossecante hiperbólica

$$\begin{array}{rccl} \operatorname{argcosech} : & \mathbb{R}\setminus\{0\} & \longrightarrow & \mathbb{R}\setminus\{0\} \\ & x & \longmapsto & \operatorname{cosech}^{-1}(x) \end{array}$$

Argumento da secante hiperbólica



Argumento da cossecante hiperbólica