Universidade do Minho
Departamento de Matemática

Algumas propriedades das funções trigonométricas

1. $\forall\, a \in \mathbb{R} \quad \operatorname{sen}^2 a + \cos^2 a = 1$

2. $\forall\, a \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{tg}^2 a = \sec^2 a$

3. $\forall\, a \in \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\} \quad 1 + \operatorname{cotg}^2 a = \operatorname{cosec}^2 a$

4. $\forall\, a \in \mathbb{R} \quad \operatorname{sen}(-a) = -\operatorname{sen} a \quad$ (sen é ímpar)

5. $\forall\, a \in \mathbb{R} \quad \cos(-a) = \cos a \quad$ (cos é par)

6. $\forall\, a \in \mathbb{R} \quad \cos(\frac{\pi}{2} - a) = \operatorname{sen} a \quad$ e $\quad \operatorname{sen}(\frac{\pi}{2} - a) = \cos a$

7. $\forall\, a \in \mathbb{R} \quad \operatorname{sen}(a + 2\pi) = \operatorname{sen} a \quad$ (sen tem período 2π)

8. $\forall\, a \in \mathbb{R} \quad \cos(a + 2\pi) = \cos a \quad$ (cos tem período 2π)

9. $\forall\, a,\, b \in \mathbb{R} \quad \operatorname{sen}(a + b) = \operatorname{sen} a \cos b + \operatorname{sen} b \cos a$

10. $\forall\, a,\, b \in \mathbb{R} \quad \cos(a + b) = \cos a \cos b - \operatorname{sen} b \operatorname{sen} a$

11. $\forall\, a,\, b \in \mathbb{R} \quad \cos a - \cos b = -2 \operatorname{sen} \frac{a-b}{2} \operatorname{sen} \frac{a+b}{2}$

12. $\forall\, a,\, b \in \mathbb{R} \quad \operatorname{sen} a - \operatorname{sen} b = 2 \operatorname{sen} \frac{a-b}{2} \cos \frac{a+b}{2}$

Algumas propriedades das funções hiperbólicas

$$\operatorname{sh} : \quad \mathbb{R} \longrightarrow \mathbb{R} \qquad \operatorname{ch} : \quad \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto \frac{e^x - e^{-x}}{2} \qquad\qquad x \longmapsto \frac{e^x + e^{-x}}{2}$$

1. $\forall\, a \in \mathbb{R} \quad \operatorname{ch}^2 a - \operatorname{sh}^2 a = 1$

2. $\forall\, a \in \mathbb{R} \quad \operatorname{th}^2 a + \operatorname{sech}^2 a = 1$

3. $\forall\, a \in \mathbb{R} \setminus \{0\} \quad \operatorname{coth}^2 a - \operatorname{cosech}^2 a = 1$

4. $\forall\, a \in \mathbb{R} \quad \operatorname{sh}(-a) = -\operatorname{sh} a \quad$ (a função sh é ímpar)

5. $\forall\, a \in \mathbb{R} \quad \operatorname{ch}(-a) = \operatorname{ch} a \quad$ (a função ch é par)

6. $\forall\, a,\, b \in \mathbb{R} \quad \operatorname{sh}(a + b) = \operatorname{sh} a \operatorname{ch} b + \operatorname{sh} b \operatorname{ch} a$

7. $\forall\, a,\, b \in \mathbb{R} \quad \operatorname{ch}(a + b) = \operatorname{ch} a \operatorname{ch} b + \operatorname{sh} b \operatorname{sh} a$

8. $\forall\, n \in \mathbb{N} \quad \forall\, a \in \mathbb{R} \quad (\operatorname{ch} a + \operatorname{sh} a)^n = \operatorname{ch}(na) + \operatorname{sh}(na)$